# Fracture behaviour of composites based on $AI_2O_3$ - TiC

## R. P. WAHI,\* B. ILSCHNER

University of Erlangen-Nürnberg, Martensstrasse 5, 8520 Erlangen, West Germany

The critical stress intensity factor and other related fracture parameters have been measured in three-point bending for pure  $AI_2O_3$ -TiC composites containing 4 to 35 volume fractions of TiC. An increase has been observed for all the parameters with increasing volume fraction of TiC. Following a study of the mode of fracture, the results are explained in terms of a linear variation of the fracture energy with the volume fraction of TiC.

## 1. Introduction

It has been shown [1-11] that alloying of brittle materials, in particular ceramics, to produce a two-phase structure may result in considerable improvement in their fracture toughness. In an attempt to explain these results theoretically, Lange [12] and Evans [13] have proposed models of crack propagation in two-phase brittle materials. These models envisage the pinning of the crack front by hard second-phase particles, thereby causing an improvement in strength as well as in toughness of the alloys.

Claussen and co-workers [8, 9], on the other hand, maintain that the increase in toughness in their materials is associated with microcrack formation. The various approaches for enhancing the toughness of ceramics have recently been reviewed by Evans *et al.* [14].

The fracture behaviour of coarse-grained  $Al_2O_3$ -ceramics containing particles of a second phase ( $6Al_2O_3 \cdot BaO$ ) has been studied by the present authors [10, 11]. A good qualitative and quantitative agreement with the model of Evans was found. Measurements of the authors on very fine-grained  $Al_2O_3$  containing precipitates of  $Al_2O_3 \cdot MgO$ , however, did not show any influence of the second phase [15]. For further investigation of the fracture behaviour of fine-grained ceramics, the system  $Al_2O_3$ -TiC was chosen. Fine-grained composites based on this system possess high wear resistance and as such are used in industry as tool materials for very high-speed cutting/grinding of hard materials.

## 2. Experimental procedure

#### 2.1. Materials

The hot-pressed composites containing 5 to 40 wt % TiC (Table I) were supplied by Feldmühle AG, Plochingen, West Germany. All composites were produced under uniform conditions of mixing and hot pressing. For comparison, samples of hot-pressed  $Al_2O_3$  (W304) fabricated under identical conditions were also included in the investigations.

2.2. Microstructure and fracture parameters The porosity, P, of the composites was calculated from the measured and theoretical densities,  $\rho_{\rm E}$  and  $\rho_{\rm T}$ , respectively:

$$P = (\rho_{\rm T} - \rho_{\rm E})/\rho_{\rm T}.$$
 (1)

Here  $\rho_{\rm T}$  is the weighted average of the known theoretical densities of the two pure components  $[\rho_{\rm T} ({\rm Al}_2{\rm O}_3) = 3.986\,{\rm g\,cm}^{-3}$  and,  $\rho_{\rm T} ({\rm TiC}) =$  $4.92\,{\rm g\,cm}^{-3}$  [16].] For the microstructural examination, the specimens were diamond polished, and the pure alumina specimens were thermally etched (30 min at 1200° C). The TiC particles could be revealed under an optical microscope in the as-polished condition. The average size of the TiC-particles, the Al<sub>2</sub>O<sub>3</sub> grains and their respective distribution characteristics in the singlephase specimens were determined with the help of line intercept method [17].

The elastic modulus, E, of the composites was determined by measuring their resonance frequency, f, in a Förster Elastomat:

\*Present address: Hahn-Meitner-Institute for Nuclear Research, Glienicker Straße 100, 1000 Berlin 39, West Germany. 0022-2461/80/040875-11\$03.10/0 © 1980 Chapman and Hall Ltd. 875

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Comnositee	Tin	Density		Poroeity	Volume	Average	Average	БІÄ	Critical	Flactic	Fractu	1	Critical et	Pece	Work	Fracture
and and another	(wt%)	(g cm <sup>-3</sup>	~	P (%)	fraction	diameter	distance	1	inherent	modulus.	stress,	2	intensity f	actor	of	energy.
	fw			• •	of TiC,	of TiC	between		cracksize	E	80		KIC (MN	m <sup>-3/2</sup> )	fracture,	л л
		Ξ 2	H		fv (%)	2R (µm)	particles		(mп) в	(GN m <sup>-2</sup> )	(MN II	(-3)	K‡	Kres	γF	(J m <sup>-2</sup> )
							2ď (μm)				$\sigma_{ m B}^{*}$	o <sub>B</sub> †		- 01	(J m <sup>-2</sup> )	
W304	0	3.966	3.986	0.50	1	ł	1	1	175	393	332	305	4.10	4.55	19.4	20.4
W305	ŝ	3.998	4.033	0.87	4.1	0.89	13.89	0.064	68	399	465	462	6.20	4.29	14.7	19.8
W306	10	4.051	4.079	0.69	8.3	0.68	5.00	0.136	56	399	536	502	6.05	4.22	14.3	18.5
W307	20	4.109	4.173	1.53	16.8	1.18	3.89	0.303	59	405	318	530	6.60	4.61	14.5	21.5
W308	30	4.211	4.266	1.29	25.8	0.77	1.48	0.522	43	410	400	635	6.16	4.69	16.1	21.3
W309	40	4.304	4.359	1.28	35.1	0.97	1.20	0.811	44	415	352	670	6.30	4.99	16.5	26.5
*Ground spe †Polished sp	cimens. scimens.															
‡Machined n	lotch.															
<sup>§</sup> Natural cra	ck.															

$$E = (48l^4 \pi^2 \rho_{\rm E}/m^4 a^2) \cdot f^2.$$
 (2)

Here *l* is the length of the specimen, *a* is a specimen dimension parallel to the vibrations in the specimen and *m* is a constant [18]. The accuracy of *E*-measurements was  $\pm 1\%$ . The technique also allowed the determination of Poisson's ratio, *v*, whose values for these composites were found to lie between 0.23 and 0.25.

The fracture parameters, namely, the fracture strength,  $\sigma_{\rm B}$ , the critical stress intensity factor,  $K_{\rm IC}$ , the fracture energy  $\gamma_{\rm I}$  and the work of fracture  $\gamma_{\rm F}$  were measured on bar-shaped specimens of rectangular cross-section (2 mm × 4 mm × 32 mm). The specimens were fractured in three-point bending in a universal testing machine (Instron). For computing  $\sigma_{\rm B}$  and  $K_{\rm IC}$  from fracture load, P, and specimen geometry, the following well-known relations were used:

$$\sigma_{\rm B} = 3P1/2bw^2 \tag{3}$$

$$K_{\rm IC} = \sigma_{\rm B} \sqrt{c \cdot Y(x)} \tag{4}$$

$$\gamma_{\rm I} = (K_{\rm IC}^2/2E) (1-\nu^2). \tag{5}$$

Here *l* is the span of the specimen, *c* the notchdepth, *b* and *w* the breadth and the width of the specimen respectively, and Y(x) is a correction factor which is a function of the relative notch depth x (x = c/w). For specimens with x = 0.3, the value of Y(x) given in [19] was employed, whereas for x = 0.75 the function suggested in [20] was used, see also [21].

Measurements of  $\sigma_{\rm B}$  were carried out (a) on specimens whose surfaces were ground with a D30 diamond wheel, and (b) on specimens whose tension-side was diamond-polished to a finish of  $R_{\rm t} = 0.8$ . In both cases a constant cross-head speed of  $100\,\mu{\rm m\,min^{-1}}$  and a span of 30 mm were used. For the measurements of  $K_{\rm IC}$  the following two types of cracks were employed.

(1) A sharp natural crack produced in a separate controlled fracture experiment. To obtain this, specimens with a triangular notch [22] were fractured to the base of the notch in a controlled manner [23]. The relative notch depth was maintained in all cases at  $0.75 \pm 0.02$ .

(2) A machine-cut straight edge notch ( $x = 0.30 \pm 0.01$ ) produced with a 0.1 mm thick diamond wheel.

In the case of specimens with natural crack, the catastrophic fracture necessary for a valid determination of  $K_{IC}$  was realized by using a shorter

span (16 mm) and higher cross-head speed (500  $\mu$ m min<sup>-1</sup>). For specimens with machine-cut notch these parameters were the same as used for  $\sigma_{\rm B}$  determination.

 $\gamma_{\rm F}$  was calculated from the area under load/ deflection curve obtained in a controlled fracture experiment [23].

#### 3. Results

All the measured and derived parameters are shown in Table I. The values of the measured parameters represent an average of at least five measurements.

#### 3.1. Microstructure

The measured densities  $\rho_{\rm E}$  of the composites, shown in Table I, are plotted in Fig. 1a as a function of the volume fraction,  $f_v$ , of TiC particles. The  $f_v$  values were derived from respective weight fractions,  $f_w$ , and using theoretical densities of TiC and Al<sub>2</sub>O<sub>3</sub>. The variation of  $\rho_{\rm E}$  with  $f_{\rm v}$  is in good agreement with that of  $\rho_{T}$  with  $f_{v}$ . The calculated porosities of the composites lie between 0.5 and 1.5% (Table I). Some typical micrographs of the composites are shown in Fig. 2. The  $Al_2O_3$ grains (Fig. 2a) as well as the particles of TiC (Fig. 2b) show a log-normal distribution (Fig. 3). The average diameter,  $\overline{D}$ , of the Al<sub>2</sub>O<sub>3</sub> grains measured in W304 is  $0.83 \,\mu m$  and the average diameter,  $2\overline{R}$ , of TiC-particles varies from 0.68 to  $1.18\,\mu\text{m}$ . These data show that all the composites are dense and very fine grained. The size of Al<sub>2</sub>O<sub>3</sub> grains in composites (W305-W309) could not be revealed with sufficient clarity. However, since all the investigated materials were fabricated under identical conditions, it can be assumed that the grain size of Al<sub>2</sub>O<sub>3</sub> in the composites is about the same as that in W304.

#### 3.2. Fracture parameters

Variations in E,  $\sigma_{\rm B}$ ,  $K_{\rm IC}$ ,  $\gamma_{\rm I}$  and  $\gamma_{\rm F}$  with  $f_{\rm v}$  are represented in Figs. 1 and 4. All the parameters show a rising trend with increasing  $f_{\rm v}$ . In the range of  $f_{\rm v}$  from 0.0 to 0.35, the relative increases in E,  $\sigma_{\rm B}$  (polished specimens),  $K_{\rm IC}$  (natural cracked specimens),  $\gamma_{\rm I}$  and  $\gamma_{\rm F}$  amount to about 5%, 56%, 38%, 46% and 22%, respectively.  $\sigma_{\rm B}$ measured on ground specimens and  $K_{\rm IC}$  measured on specimens with machine-cut notch, however, do not show any influence of TiC addition.



Figure 1 Variation in (a) the density,  $\rho$ , and (b) elastic modulus, E, of composites with the volume fraction,  $f_v$ , of TiC.



Figure 2 Microstructure of the composites. (a)  $Al_2O_3$  grains in W304, (b) TiC particles in W307 ( $f_v = 0.17$ ). 878

Figure 3 Distribution of grain/particle diameter.



#### 4. Discussion

#### 4.1. Fracture strength

The strength of brittle materials is known [24] to be markedly influenced by the nature of their surface preparation. Surface defects introduced during grinding can drastically reduce the fracture strength. In the present specimens the lowering of  $\sigma_{\rm B}$  in all the composites caused by the grindinginduced surface defects is obviously large enough to more than compensate for the smaller increase due to TiC addition. As a result, the  $\sigma_{\rm B}$  values do not show any variation with  $f_{\rm v}$ . In contrast to this behaviour, the influence of TiC addition on  $\sigma_{\rm B}$ is clearly brought out when the extrinsic effect due to surface damage is reduced to a minimum by polishing the tension side of the bend specimens.

The expressions for  $\sigma_{\rm B}$  and  $K_{\rm IC}$ , according to Griffith and Irwin, respectively, are:

$$\sigma_{\rm B} = \frac{1}{Y(x)} \left( \frac{2E\gamma_{\rm I}}{c \cdot (1-\nu^2)} \right)^{1/2} \tag{6}$$

$$K_{\rm IC} = \left(\frac{2E\gamma_{\rm I}}{1-\nu^2}\right)^{1/2} \tag{7}$$

$$\sigma_{\rm B} = \frac{K_{\rm IC}}{y(x) \cdot \sqrt{c}} \,. \tag{8}$$

According to Equation 8, the observed variation in fracture stress with the volume fraction,  $f_v$ , of TiC must be due to variations in  $K_{IC}$  and flawsize, c, with  $f_v$ . Since the total fractional increase in  $\sigma_B$  (56%) is larger than the total fractional increase in  $K_{IC}$  (38%), a part of the increase in fracture stress must be done to corresponding decrease in flaw size with  $f_v$ . A rough measure of the notch depth, c, is given by  $(K_{IC}^2/\sigma_B^2) \cdot (\pi/4)$ . Fig. 5 shows a plot of the computed values of  $\sqrt{c}$ as a function of  $f_v$ . As expected from Figs. 4a to c, the value of  $\sqrt{c}$  falls with increasing  $f_v$ . Dispersion of TiC particles appear to restrict the size of inherent flaws in the composites and thereby further contributes to their strength.

#### 4.2. Stress intensity factor

or

For valid  $K_{IC}$  measurements on ceramics using notched specimens, it is essential to keep the notch-root radius (= 1/2 notch width) smaller than a critical value which is a material parameter





Figure 4 Continued



Figure 5 Variation in the inherent crack size c with  $f_{v}$ .

[25, 26]. Use of notches of larger width results in an over-estimation of  $K_{IC}$  which increases with notch width [26]. In case of fine-grained Al<sub>2</sub>O<sub>3</sub> it has been shown [25, 27] that a notch width of  $\leq 0.1$  mm is necessary for valid  $K_{\rm IC}$  measurement. Our results on hot-pressed Al<sub>2</sub>O<sub>3</sub> (W304) using a machined notch of 0.1 mm width confirm this. The measured value of  $4.1 \text{ MN m}^{-3/2}$  is in good agreement with  $K_{IC}$  value for  $Al_2O_3$  of comparable porosity measured in [25], using double cantilever beam specimens. In case of the composites, however, the critical notch root-radius appears to be smaller than 0.05 mm employed in the present investigations. The apparent high values of  $K_{\rm IC}$  ( $\simeq 6 \,{\rm MN \, m^{-3/2}}$ ) measured in all the composites using notched specimens therefore do not allow the influence of TiC additions to be seen (Fig. 4b). However, when the effect of notch root-radius was eliminated by using specimens with sharp natural cracks, a small but distinct influence of TiC addition on  $K_{IC}$  could be detected.

In terms of the model of Evans the expected increase in  $K_{IC}$  as a function of the parameter  $\overline{R}/\overline{d}$  can be computed using Equations 18 and 19 of Evans [13] and taking the  $K_{IC}$  value of Al<sub>2</sub>O<sub>3</sub>





to be  $4 \text{ MN m}^{-3/2}$ . The computed values of  $K_{\text{IC}}$ corrected for crack interaction at the back of the particles are shown as a function of  $\overline{R}/\overline{d}$  in Fig. 6. For comparison, the measured values of  $K_{IC}$  are also included in the same diagram. The average spacing 2d between the particles in the present composites was calculated from the respective  $f_{\rm v}$  and  $\overline{R}$  values according to Fullman [28]. It is evident that the experimentally determined rise in  $K_{IC}$  as a function of  $\overline{R}/\overline{d}$  is much smaller than the increase expected in terms of the theoretical model. Moreover, the fracture surface of the composites does not show the characteristic feature expected on the basis of the crack-pinning model. As a consequence of the bowing of crack front between the particles of second phase and a reunion of the two branches of the crack front on the back side of the particles, wedge-shaped cleavage steps are expected to form on this side of the particles. Such steps have been observed by Lange [1] and by Wahi et al. in Al<sub>2</sub>O<sub>3</sub>-BaO alloys [11]. On the fracture surface of the present composites, these steps could not be detected. Instead, an examination of the fracture surface showed that TiC particles mostly had a trans-

crystalline fracture. This observation is in agreement with the result of Chermant *et al.* [29] who have reported a predominantly transcrystalline fracture in pure TiC. Some typical fractographs are shown in Fig. 7. TiC particles with fine cleavage steps running through the grains are clearly to be seen. The particles could be identified by means of EDAX analysis of the fracture surface.

If the crack front cuts through the particles instead of bypassing them, the average fracture energy,  $\gamma_{\rm I}$ , of the composites should, to a first approximation, lie between the fracture energies of Al<sub>2</sub>O<sub>3</sub> and TiC and vary linearly with the fraction of the total fracture surface,  $f_{\rm s}$ , occupied by TiC. Now turning to Equation 7, *E* has been found experimentally to vary linearly with  $f_{\rm v}$ (Fig. 1b). If  $\gamma_{\rm I}$  were also to be a linear function of  $f_{\rm v}$  (according to [25]  $f_{\rm s} = f_{\rm v}$ ), the expression for  $K_{\rm IC}^2$  from Equation 7 could be written as follows:

$$K_{\rm IC}^2 = 2\{(E_2 - E_1) \cdot f_{\rm v} + E_1\} \\ \times \{(\gamma_2 - \gamma_1) \cdot f_{\rm v} + \gamma_1\}$$
(9)

or 
$$K_{\rm IC}^2 = 2\{(\Delta E \cdot \gamma_1 + \Delta \gamma \cdot E_1) \times f_{\rm v} + E_1 \cdot \gamma_1 + \Delta E \cdot \Delta \gamma \cdot f_{\rm v}^2\}.$$
 (10)



Figure 7 Some typical fractographs. (a) W307 ( $f_v = 0.17$ ), (b) W309 ( $f_v = 0.35$ ).

Neglecting the term  $\Delta E \cdot \Delta \gamma \cdot f_v^2$  which is small compared to the rest of the terms,  $K_{IC}^2$  can be approximately written as a linear function of  $f_v$ :

$$K_{\rm IC}^2 = 2\{(\Delta E \cdot \gamma_1 + \Delta \gamma \cdot E_1) \cdot f_{\rm v} + E_1 \cdot \gamma_1\}.$$
(11)

Here  $\Delta E$  and  $\Delta \gamma$  are the differences between the *E*-values and  $\gamma_{\rm I}$  values of Al<sub>2</sub>O<sub>3</sub> and TiC, respectively, and  $E_1$  and  $\gamma_1$  are the elastic modulus and fracture energy, respectively, of Al<sub>2</sub>O<sub>3</sub>.

Fig. 8 shows a plot of  $K_{IC}^2$  as a function of  $f_v$ . The experimental points lie fairly close to the straight line drawn on the basis of least squares fit. An extrapolation of the straight line to 100% TiC leads to a  $K_{IC}^2$  value of 40.10 MN<sup>2</sup> m<sup>-3</sup> which corresponds to a  $K_{IC}$  of 6.33 MN m<sup>-3/2</sup> for pure TiC.

Mai [30] has measured  $K_{IC}$  of dense TiC (employing specimens with natural crack) to be 7.5 MN m<sup>-3/2</sup>. In view of the simplifying assumptions made in arriving our Equations 11 and the large extrapolation of the experimental values involved, a difference of about 15% between the two values of  $K_{IC}$  appears to be small. Two other independent measurements of  $K_{IC}$  on TiC, on the other hand, report values of 3.8 MN m<sup>-3/2</sup> [29] and  $3.9 \,\mathrm{MN}\,\mathrm{m}^{-3/2}$  [31] which do not fit in the above model. The density of TiC in [29], however, appears to be smaller than that of our materials. Moreover, the nature of the crack employed in [29] is not clear. Both these factors can considerably influence the  $K_{IC}$  value. The measurements in [31] are made on dense TiC using natural crack, as in the present work.



Figure 8 Variation in  $K_{IC}^2$  as a function of  $f_v$ .

Let us now consider the possible influence of microcracks on the fracture behaviour. It has been shown [32] that microcracks may form in twophase brittle materials as a result of different coefficients of linear thermal expansion,  $\alpha$ , of the matrix and the second phase. The cracks would form provided the second-phase particles are larger than a critical size,  $R_c$ , which is inversely proportional to the square of the difference  $\Delta \alpha$  in the two  $\alpha$  values. In ceramics such as Al<sub>2</sub>O<sub>3</sub> containing  $ZrO_2$  where considerable improvement in fracture toughness has been achieved through microcracks [8, 9], the effective value of  $\Delta \alpha$  is quite large  $(2.8 \times 10^{-5} \circ C^{-1})$ . This means that even very small particles of  $ZrO_2$  ( $\simeq 1 \,\mu m$ ) are able to initiate cracks. The toughness can be improved therefore, through uniformly distributed fine microcracks. In the present composites, however,  $\Delta \alpha$  amounts only to  $5.6 \times 10^{-6} \circ C^{-1}$  $[\alpha(Al_2O_3) = 8 \times 10^{-6} \circ C^{-1}$  and  $\alpha(TiC) = 2.4 \times 10^{-6} \circ C^{-1}$  $10^{-6} \circ C^{-1}$  [16]]. The value of  $R_c$  for TiC particles can be calculated using the Equation 12 in [32]. Assuming the parameter  $H(\mu p)$  in the above equation to be unity and substituting values for the other parameters from the present investigation (E for TiC from [31] and the  $\alpha$ -values from [16]) one obtains  $R_c \simeq 4 \,\mu m$ . Thus it appears that microcracks may not play an important role in the present composites, since the average size of the particles is much smaller (0.34 to  $0.59 \,\mu\text{m}$ ). However, unless  $H(\mu_0)$  is correctly known (it is likely to be smaller than unity) the possibility of a microcrack influence cannot be completely ruled out.

#### 5. Conclusions

The fracture behaviour of  $Al_2O_3$ -TiC ceramics containing a wide range of volume fraction of TiC has been investigated. The study of the dependence of fracture parameters as a function of the volume fraction,  $f_v$ , of TiC has lead to the following conclusions:

(1) addition of TiC results in a small but distinct increase in  $K_{IC}$ ,

(2) the observed increase in  $K_{IC}$ , within the range of investigated composites, shows a good agreement with the proposed model of linear dependence of  $K_{IC}^2$  on  $f_v$ ,

(3) use of a machined notch of 0.1 mm thickness in three-point bent specimens of the composites results in an overestimating of  $K_{IC}$ ,

(4) the size of the inherent flaws in the com-

posites appears to depend on  $f_v$  – decreasing with increasing  $f_v$ . The fracture stress, therefore, shows a stronger dependence on  $f_v$  than shown by  $K_{IC}$ .

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